Bidding in Hierarchies*

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Abstract

This paper reconsiders the comparison between hierarchical contests and single-stage contests. A condition is given that characterizes whether and when the aggregate equilibrium payoff of contestants is higher in the single-stage contest, and when the single-stage contest is more likely to award the prize to the contestant who values it most highly. The outcome depends on inter- and intra-group heterogeneity, and is not driven by free-rider incentives.

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1 Introduction

If the winner of a contest enters another contest in which the prize allocated in the previous contest is again contested, a problem with respect to a contestant's willingness to invest in winning the first contest that is similar to the hold-up problem in investment decisions is generated. The incentive to compete for the prize in the early round is reduced by the fact that the winner now enters another contest and has to spend further resources before winning anything. Various applications such as Wärneryd (1999), who considers resource allocation within federations, Inderst, Müller and Wärneryd (2002), who consider the allocation of free cash flow inside organizations, and Müller and Wärneryd (2001), who consider distributional conflict between shareholders of corporations, all draw on a particularly interesting structure that is as follows. There are two contest stages. In an inter-group contest, several groups of individual contestants first compete for a prize. Once the prize is shared out among these groups, the individual members of a group compete in an intra-group contest for receiving what the group gained in the inter-group contest. The central result is that the coordination problem within members of groups in the inter-group contest and the problem that the group prize will be subject to the future within-group contest can reduce total contest effort. This suggests that, compared to a big single-stage contest in which all individual contestants compete with each other directly, a hierarchical structure can reduce wasteful contest effort.

The result is an important contribution to the recent literature on endogenous property rights that is outlined in Skaperdas (2003). It applies to situations in which the allocation of resources is not well determined by costlessly enforceable property rights and suggests that there is a motivation for generating hierachical structures through which the resources are channelled. The analysis is, however, limited to contest success functions for which the probability of winning is a continuous function of efforts and is typically measured by the ratio of own effort to aggregate effort, like in the ratio function introduced by Tullock (1980) for rent-seeking contests, and the result is obtained where contestants are homogenous, or for some moderate heterogeneity.

In this paper I consider a contest success function through which the contestant who makes the highest effort wins with certainty[1] and I will focus on the role heterogeneity within, and across, competing groups.[2] It turns out that the key result in the abovementioned work may reverse: depending on the heterogeneity within and between groups, a hierarchical structure may increase the degree of rent dissipation. Moreover, contests are also used consciously as devices to identify the most productive contestant or the contestant who values the prize most highly. I find that hiearchies will typically reduce the quality of a contest as a mechanism for this purpose and may single out contestants who have a very low valuation of the prize.

A related literature also considers an inter-group contest for a prize, and the problem of allocating the prize within the winning group. Unlike the considerations here, however, the allocation of the prize is determined by exogenous rules, and not as the outcome of an intra-group contest following the inter-group contest (see, e.g., Katz, Nitzan and Rosenberg 1990, Nitzan 1991a, 1991b, Davis and Reilly 1999). Most of this literature considers a Tullock (1980) contest success function. A recent paper by Baik, Kim and Na (2001) reconsiders the questions that are addressed in Katz, Nitzan and Rosenberg (1990) for a contest success function without noise. In some sense, our paper also complements the analysis by Baik, Kim and Na (2001), using their contest success function. Baik, Kim and Na (2001) consider a prize that is a public good for the whole group that wins the prize. Hence, there is no need to consider the intra-group allocation of the prize. I consider a prize that is not a public good within the group and assume that there is an intra-group contest for the allocation of the prize, once the group obtained the prize in the inter-group contest.

[1] This contest success function has also received a considerable amount of attention in the literature and in many areas of application. For some examples and further references see, e.g., Hirshleifer and Riley (1992), Hillman and Riley (1989), Ellingsen (1991), Baye, Kovenock and deVries (1996), Che and Gale (1998) and Konrad (2002).

[2] The strong disincentives to spend effort in an all-pay auction if the prize of winning must be defended in a later contest have already been discussed in a framework with symmetric firms and a consumer group bidding in the rent-seeking for monopoly framework by Ellingsen (1991).

2 The analysis of hierarchy

Suppose there are n groups that constitute the set $N = \{1, ...n\}$. Each group $i \in N$ consists of $m_i \geq 1$ members that constitute the set M_i. In a first stage the groups compete in what is called the inter-group contest for a given prize. Group members make contributions to a group's effort in winning the prize, with $x_{ij} \geq 0$ the effort by member j of group i. Efforts of group members sum up to the group's total effort $x_i = \sum_{j=1}^{j=m_i} x_{ij}$. The group that exhibits the highest aggretate x_i wins the prize. More precisely, let L be the set of groups $l \in N$ with $x_l \geq x_r$ for all $r \in N$. Then the probability p_i that group i wins is zero if $i \notin L$ and equal to $1/(\#L)$ if $i \in L$, where $\#L$ denotes the number of elements of L.

Once the prize is allocated to one of the groups, the members within this group compete for the prize in an intra-group contest that is structurally the same as the one among groups. Let group i win the prize. Then each member j chooses some $y_{ij} \geq 0$. Now let L_i be the set of individuals in group i with $y_{il} \geq y_{ir}$ for all $r \in M_i$. Then the probability q_j that member j wins is zero if $j \notin L_i$ and equal to $1/(\#L_i)$ if $i \in L_i$. Note that y_{ij} is not a function of the person's contribution x_{ij} to group effort: it is the equilibrium effort chosen in the subgame described by the intra-group contest.

Finally, individuals may differ with respect to their valuation of the prize.[3] Let u_{ij} be the value that player j in group i attributes to winning the prize. Let the members of each group be sorted according to their valuations of the prize: $u_{ij} > u_{i(j+1)}$ for all i and all j. To simplify the exposition, let these inequalities hold strictly. The size of $u_{i1} - u_{i2}$ for a group i will be called this group's 'heterogeneity at the top' and will play a central role in what follows.

Consider the equilibrium in the continuation game that emerges once a group i has been determined as the winner of the prize. The payoff of contestant j in the group i in case i wins the prize is

$$v_{ij} = q_j(y_{i1}, ...y_{im_i})u_{ij} - y_{ij} \text{ for all } j \in M_i.$$

[3] A structurally equivalent way to describe the problem would be to consider contestants who all have the same valuation of the prize, but differ in their unit cost of effort (see Baye, Kovenock and deVries 1996).

The contest among group members in this case has a well-known unique equilibrium outcome that has been described in detail by Baye, Kovenock and deVries (1996): only the contestants $i1$ and $i2$ who are the contestants with the highest and second highest u_{ij}, respectively, expend effort and randomize their efforts according to cumulative distribution functions of effort that are described as follows:

$$F_{i1}(y_{i1}) = \frac{y_{i1}}{u_{i2}} \text{ for } y_{i1} \in [0, u_{i2}] \tag{1}$$

and

$$F_{i2}(y_{i2}) = (1 - \frac{u_{i2}}{u_{i1}}) + \frac{y_{i2}}{u_{i1}} \text{ for } y_{i2} \in [0, u_{i2}] \tag{2}$$

and $F_{ij}(y_{ij}) = 1$ for $y_{ij} \geq u_{i2}$ for $i = 1, 2$. Hence, contestant $i1$ has a payoff equal to $(u_{i1} - u_{i2})$ and all other contestants have zero payoff. Uniqueness holds if $u_{i2} > u_{i3}$ as was assumed here. (See Baye, Kovenock and deVries (1996) for a proof).

Only one player in each group has a net benefit from making his group win the contest. For group i this is the group member with the highest u_{ij} in this group. By an appropriate numbering of group members, this was member $j = 1$ in each group. Hence, this contestant $i1$ in group i attributes a value to the outcome of his group winning the prize equal to $(u_{i1} - u_{i2})$. Only this player will make contributions; accordingly, the group contributions to the contest between groups are $x_i = x_{i1}$. This makes the problem of finding the equilibrium in the contest between groups equivalent to a contest between n contestants $i1$, one from each group i.

The payoff functions of these n contestants can be stated as

$$v_{i1} = p_i(x_1, ...x_n)(u_{i1} - u_{i2}) - x_i.$$

The equilibrium for the case of n contestants with these objective functions is again the one for the standard all-pay auction as in Baye, Kovenock and deVries (1996). Consider the numbering of groups (re-) numbered according to their heterogeneity at the top, such that $u_{i1} - u_{i2} \geq u_{(i+1)1} - u_{(i+1)2}$ for all $i = 1, ...(n-1)$ and assume that $u_{21} - u_{22} > u_{31} - u_{32}$, that is, the inequality holds strictly with respect to the two groups for which this term is second and third largest.[4] Then this contest again has a unique equilibrium and this equilibrium is described as follows. All $i1$ with $i > 2$ spend $x_i = 0$. The active contestants in group 1 and 2 choose mixed strategies that are described by cumulative density functions of x_1 and x_2 with

$$G_1(x_1) = \frac{x_1}{u_{21} - u_{22}} \text{ for } x_1 \in [0, u_{21} - u_{22}]$$

and

$$G_2(x_2) = \left[1 - \frac{u_{21} - u_{22}}{u_{11} - u_{12}}\right] + \frac{x_2}{u_{11} - u_{12}} \text{ for } x_2 \in [0, u_{21} - u_{22}],$$

and $G_1(x) = G_2(x) = 1$ for $x > u_{21} - u_{22}$. Accordingly, the payoff of all contestants except member 1 of group 1 is zero, and the payoff of this contestant is equal to

$$v_{11}^* = (u_{11} - u_{12}) - (u_{21} - u_{22}). \tag{3}$$

This is summarized as a proposition.

Proposition 1 *The equilibrium payoffs in the two-stage contest are zero for all players except for player* 1 *in group* 1*. This player has an equilibrium payoff* v_{11}^* *as in (3). The prize is allocated to four players with positive probability: players* 1 *and* 2 *in groups* 1 *and* 2.

In the contest the maximum payoff for the contestants is obtained if the bidder with the highest valuation receives the prize and no contestant makes a bid. In the equilibrium this is not the outcome because the contestants spend positive amounts of effort and because the prize does not necessarily end up with the contestant with the highest valuation of the prize. Hence, from the contestants' perspective, there is some dissipation of the prize. Proposition 1 shows that the amount dissipated is smaller if player 1 in group 1 values the prize much more highly than other members in his group, and if the difference in valuation is smaller in the only group that actively competes, which, by appropriate numbering of groups, is group 2, and if the difference

[4]This assumption avoids a multiplicity of equilibria in the all-pay auction that is discussed more fully in Baye, Kovenock and deVries (1996).

in valuations of the prize for players 1 in the groups 1 and 2 is high. The stage-two contests between members of the winning group dissipate all rent except for the payoff-difference between the contestants whose valuation of the prize is highest within the respective group, and this remaining rent goes to the group member whose valuation of the prize is the highest. This explains why only they make contributions to the group efforts, and why no one else participates in the attempt to make the own group win. Hence, the contest between groups is essentially a contest between these single members who value the prize most highly within their groups, one member from each group. These members' stakes in trying to get the prize allocated to their own group are determined by their lead in their respective within-group contest. This lead generates the rent they can appropriate if the prize is awarded to their group.

3 Flat or steep hierarchies?

The outcome in the two-stage contest maps the situation in a hierarchy in which the prize is first allocated among several groups (the upper layer of the hierarchy, e.g., the states in a federation, or large organizational units in firms) and then allocated among the members of the group that wins the first contest (e.g., interest groups within the state that wins the prize, or sub-units of the organizational unit in the firm that wins the prize). To see whether hierarchies are advantageous for reducing total contest effort in an organization and for allocating the prize to the agents who value it most, this outcome must now be compared with the situation in which a single stage contest among all contestants represents the situation without a hierarchy.

Baye, Kovenock and deVries (1996) can again be used to describe the outcome of the big single-stage contest. All contestants receive a payoff equal to zero, except for the contestant whose valuation of the prize is highest. This agent receives a rent that is determined by the difference between his own valuation and the second highest valuation. Let $\{...u_n...\}$ be the set of valuations of the prize for all individuals, the same individuals who were allocated among differnent groups in the hierarchical two-stage contest. Let u_f and u_s be the "first" and "second" highest valuation of the prize, respectively. Then this individual f's equilibrium payoff is equal to

$$v_f^* = u_f - u_s. \tag{4}$$

In general, it is not clear whether the payoff v_f^* exceeds, or falls short of, v_{11}^*. It is not even clear whether f and s coincide with players 1 in the respective groups 1 and 2. Comparing (4) with (3) immediately yields

Proposition 2 *The aggregate equilibrium payoff for the contestants is higher (dissipation is lower) in the single-stage contest if*

$$u_f - u_s > u_{11} - u_{12} - (u_{21} - u_{22}). \tag{5}$$

For the aggregate equilibrium payoff of contestants in the two-stage game the heterogeneity within groups at the upper end of the distribution of prize valuations matters, but only for the groups with the largest heterogeneity at the upper end of prize valuations. Groups were sorted by the amount of heterogeneity at the upper end, i.e., by the difference in valuation between the contestants with the two highest valuations within each group. Whether hierarchies improve the contest outcome from the perspective of the contestants as a group depends on how heterogenous the contestants are, and how they are allocated between the groups.

The contestants f and s need not belong to the groups 1 and 2. However, because $u_f = u_{i1}$ for some group i must hold, and $u_{i2} \leq u_s$ in this group, it follows that

$$u_{11} - u_{12} \geq u_f - u_s \tag{6}$$

where equality can hold only if f and s both belong to group 1. This fact makes the most heterogenous group 1 at least as heterogenous as the group of all contestants, and, taken in isolation, this reduces rent dissipation as can be shown from (3). However, whether this heterogeneity is sufficient to make rent dissipation lower in the two-stage game than in the single-stage game depends on the heterogeneity of the second-most heterogenous group, and this heterogeneity increases rent dissipation.

For instance, if f and s belong to group 1, they must be identical with group members 1 and 2 in this group 1. Therefore, applying (5) yields that the two-stage contest cannot have a lower dissipation than the single-stage contest if f and s belong to group 1.

Another example may illustrate the opposite case in which rent dissipation in the two-stage game is much lower than in the one-stage contest. Let there be four contestants with valuations $v_f = 1002$, $v_s = 1001$, $v_3 = 1000$ and $v_4 = 1$. In the single-stage game, contestants' aggregate rent equals 1002-1001=1. Now let players f and 4 be in group 1 and let players s and 3 be in group 2 and consider the two-stage game. Now the aggregate rent is equal to $(1002 - 1) - (1001 - 1000) = 1000$.

I now turn to the selection properties of the two-stage contest. In some contexts, contests or tournaments are used to select candidates for a task, and the organizer of the contest would like to allocate the task to the person who benefits most from it, or has the lowest cost of performing the task. It has been argued that, despite their second-best nature, contests may be used as selection devices if the contestants know each other's valuation or ability, but the contest organizer does not. The question is then whether the two-stage contest is more likely to allocate the prize to the contestant who values it most highly. The single-stage contest performs very well: with probability 1 the prize is obtained by one of the contestants who value the prize most highly, and the larger the difference in their valuation of the prize, the more likely it is that the contestant who has the highest valuation obtains the prize.

The two-stage contest performs worse:

Proposition 3 *(i) In the two-stage game the probability that the contestants with the two highest valuations of the prize win the prize is always smaller than 1. (ii) For any distribution of prize valuations, an allocation of contestants among groups exists such that the contestant with the lowest valuation of the prize can win the prize with positive probability. (iii) For some set of contestants and some distribution of contestants between groups the contestants with the two lowest valuations can win the prize with a probability that is arbitrarily close to 1.*

Proof. Consider (i). In the two-stage game four contestants 11, 12, 21 and 22 can win the prize with strictly positive probability. Accordingly, the probability that any two of them wins the prize is smaller than 1. Note that contestants f and s need not even belong to this group. Consider (ii). Let $u_{\min}$ be the smallest valuation. Now allocate the contestants such that two contestants with valuations $u_{11} = u_f$ and $u_{12} = u_{\min}$ constitute one group. This group will be group #1, and the contestant with valuation $u_{\min}$ will win with positive probability equal to $\frac{u_{\min}}{2u_f}(1-\frac{u_{21}-u_{22}}{2(u_f-u_{\min})}) > 0$. The proof of (iii) is by way of an example. Let there be four contestants. Suppose, for instance, that $u_{21} = u_{22} + \epsilon$ and $u_{11} = u_{12} + \Delta < u_{21}$. Then the prize is allocated to group 1 with a probability that converges towards 1 as $\epsilon \to 0$, and once the prize is allocated to group 1, it goes to player 1 or player 2 in this group with positive probabilities. But both value the prize less than the two contestants who have the highest valuation of the prize among all players in group 2. The example can be generalized to large sets of contestants provided that the groups $i = 2, 3, ...n$ are all sufficiently homogenous at the top. □

For illustration, suppose, for instance, that there are only two groups 1 and 2, and two contestants in each group, with $u_{11} = 2$, $u_{12} = 1$, $u_{21} = 1000$ and $u_{22} = 1000 - \epsilon$ with $\epsilon \in (0, 1)$. As ϵ converges towards 0, the probability that group 1 wins converges towards 1, implying that the prize goes to one of the contestants in group 1 who has a very low valuation of the prize. Intuitively, the bids that contestants make in the contest between groups depend only on the rent a contestant will obtain if his group wins. This rent does not depend on a contestant's absolute valuation of the prize, but only on the difference between his valuation and the valuation of other members of this contestant's group. In the numerical example, this rent will be equal to $2-1$ for contestant 11 and equal to ϵ for contestant 21, and zero for all other contestants. Hence, only contestants 11 and 21 will make positive bids in the inter-group contest. But if ϵ is small, 21 will bid very little, even though his absolute valuation of the prize is much higher than that of contestant 11. Accordingly, the prize is very likely to go to group 1, the group with contestants who both have comparatively low valuations of the prize.

4 Conclusions

It has been pointed out in the literature that multi-stage contests for prizes in which groups compete for a prize first, and then the prize is allocated among the members of the winning group by a second contest, can reduce total rent dissipation because the repetition of conflict generates a hold-up problem, and a free-rider problem in the contest between groups. In this paper, I analysed whether these results are robust to asymmetries and to different contest technologies. I find that hiearchies and the multi-stage contests they may generate need not do well. They may cause total effort to be higher and result in a more inefficient allocation of the prize.

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