Orchestrating Rent Seeking Contests

Mark Gradstein and Kai A. Konrad*

December 21, 1998

Abstract

Contests have different, sometimes quite complex, organizational structures. In particular, while most of the existing literature focusses on simultaneous contests, multistage contests are also quite frequently encountered. This paper seeks to provide a rationale for the latter by endogenizing the choice of a contest structure, which is made by an organizer of a contest interested in maximizing the efforts expended by the contenders.

1 Introduction

Contexts as seemingly diverse as economic organizations, sport competitions, wars, competition for natural monopoly, patent races and political rent seeking can be described as contests whereby contestants expend resources to win a prize. A classical example is the natural monopoly setting. The opportunity of making monopolistic profits invites competitive rent seeking endeavors on the part of the potential monopolists each seeking to ensure the privileged position in the industry for itself by influencing the politicians. The literature on rent seeking (see Nitzan 1994 for a survey) studies these situations focusing in particular on the relationship between the outlay made in the

*Address correspondence to: Mark Gradstein, Department of Economics, University of Pennsylvania, Philadelphia, PA 19104, e-mail: grad@ssc.upenn.edu, or Kai A. Konrad, Department of Economics, Free University of Berlin, Boltzmannstrasse 20, D-14195 Berlin, e-mail: kai.konrad@wiwiss.fu-berlin.de. The authors are indebted to two referees and the editor of this Journal for their valuable comments and suggestions.

course of such competition on the one hand and the rent at stake on the other hand.[1]

Contest design, that is the set of rules that define the victorious contestant(s), clearly has incentive effects as far as the amount of effort expended by the contestants goes. The interest in the amount expended by the rent seekers to win the rent relative to the value of the rent itself naturally begs the question of design of rent seeking contests.[2] For one thing, according to the dominant view in the rent-seeking literature, the politicians who allocate rents value high outlay and had they some freedom of choice to design a contest, they would choose the one that results in its maximization. From a normative perspective, therefore, studying contest design may help to indicate which constraints, if any, should be imposed on the ability to design a contest - indeed, in reality such constraints are often imposed either at the constitutional or at the legislative stage of the political process. From a positive perspective, the particular contest used by a politician to allocate a rent among rent seekers is itself the outcome of a political process in the course of which the bureaucrat - who will benefit from future rent seeking - would lobby for a design that leads to a larger rent-seeking effort. Our paper suggests the kind of a contest this should be.

Indeed, some work has been done on contest design in the last decade. Appelbaum and Katz (1987) and Michaels (1989) are early contributions to the issue; more recently, Baye et al. (1993) have studied optimal admittance to a contest; Lazear (1996: 79-83) discusses the use of multiple prizes in a contest where contestants differ in their contest abilities; and Dasgupta and Nti (1998) have focused on the possibility of an organizer of a contest to retain part of the prize. All these papers, however, only study simultaneous

[1] Being an ubiquitous social phenomenon, contests have been extensively studied in a variety of other contexts as well, including labor market reward structures (see e.g., Rosen, 1986), conflict theory (Hirshleifer, 1991, Skaperdas 1992, 1995), and political competition (Hillman and Ursprung 1988, Skaperdas and Grofman 1995).

[2] Tullock's (1980) seminal paper that predicts the amount expended in a contest to be almost equal the value of the rent has provoked much research interest. Different comparative statics results pertaining to the contestants' outlay have been derived in the literature since then. For instance, it turns out that the outlay is larger when the contestants are similar (with regard to their abilities, tastes, etc.) and if their number is big (Hillman and Riley 1989; Gradstein 1995). For the effect of risk aversion see Hillman and Katz (1984) and Konrad and Schlesinger (1997). Total outlay is smaller when the contestants first choose an order of play and then carry out the actual rent seeking game (see Morgan, 1998, and references therein); or when collective benefits exist (Nitzan, 1991).

(S-) contests letting the organizer choose different parameters within this framework. In contrast, here the structure of matches in the contest itself is the choice parameter, so that the winners from each stage of the overall contest are promoted to the next stage. In reality, contests exhibit various structures. For instance, while in some sport disciplines (formula I races, or long jump), we observe simultaneous contests, in other disciplines, such as tennis championships or soccer world cups, the contestants compete in multistage contests which consist of a series of pairwise contests - these will be labelled T-contests, as their structure resembles a tree.

Sometimes the contest structure is exogenously given. Our framework may then be used to predict in which type of contest efforts are larger - the examples of potential applicability include R&D contests and contests that are caused by technological change.[3] In other applications the contest structures are the outcome of a careful design with the view of attaining a variety of objectives, one of which is maximization of efforts by contenders. This is certainly the case in sports, but also in other settings as well. Internal organizational design is one case in point.[4] Indeed, Keren and Levhari (1983) and Sah (1991), the former arguing from the information-theoretic perspective and the latter in the decision-theoretic framework, have shown that a multistage organizational structure may well be optimal. We suggest yet another view of organizations, according to which one can interpret the organization's hierarchy in a steady state as consisting of a series of contests among the individual members of each level of the hierarchy for the promo-

[3] For instance, in oligopolistic competition, technological change may change the equilibrium structure in an industry and in particular decrease the equilibrium number of firms. A series of such changes may cause a stepwise reduction in the equilibrium number of firms. The process of adjustment to the new equilibrium industry structure often has the feature of a contest: firms make sunk investments in R&D and/or in excess capacity (Thum 1995) in order to increase their chances to survive, and these costs cannot be recovered even if the firm has to exit. The contest nature of dynamic competition has been recognized in the R&D literature, and in the literature on network externalities (e.g., Besen and Farrell 1994). If the technology shocks are relatively small, we observe a series of contests with the winning firms in each stage qualifying for the next contest stage - the structure is similar to a T-contest. With drastic shocks, the number of contest stages is much smaller and, if one big shock yields the new long term industry equilibrium structure, the contest resembles an S-contest.

[4] Likewise, in political situations, multistage contests are very common, either explicitly as in the case of candidate selection in the primaries and the subsequent presidential elections, or implicitly, when only individuals elected for an office are qualified to run for a higher office.

tion to an upper level. Which hierarchical structure then is the one that induces the maximal overall effort across these contests?[5]

Structural design that maximizes overall contest effort is the main focus of this paper.[6] Our results indicate that the optimal contest structure hinges on how discriminatory the contest is - that is, on the relative importance of contestants' effort to win the prize versus random factors. When the contest rules are discriminatory enough, and only then, does the S-contest elicit more effort; otherwise, the T-contest elicits more effort. An interesting corollary of our analysis implies that when contest structure can be optimally adjusted, then as the number of contenders grows large, the amount of exerted effort equals the value of the assigned prize. The rent is 'fully dissipated'. This is in stark contrast to the simultaneous contest case which has been extensively studied in the rent-seeking literature, where the sum of contest efforts is equivalent to the prize only when the contest is sufficiently discriminatory. Thus, our result that effort equal to the value of the contested rent can be elicited for any discriminatory extent of the contest technology contributes to the intense debate in the rent seeking literature on the ratio between equilibrium contest effort and the value of the rent in contests that originated with Tullock's (1980) paper.

[5]In the related context of internal labor markets, the literature on tournaments (see e.g., Lazear and Rosen, 1981, and Green and Stokey, 1983) has examined the optimal incentive schemes to ensure a maximal effort. In contrast to the rent seeking models, this literature typically invokes the assumption that the amount to be divided among the contestants is affected by their effort, which is non-observable or non-verifiable.

[6]Three other contributions belong to this research agenda. Lazear (1996: 127-133) considers two-stage contests that differ with respect to the number of matches played. In particular, he compares knock-out systems and all-play-all contests. The number of contest stages is exogenous, but contestants differ with respect to ability. Gradstein (1998) compares simultaneous and pairwise multistage contests with a specific probability-of-winning function addressing the issue of timing of effort exertion which is absent in this paper, but without taking up the general problem of contest design, namely, what contest structure is optimal when the class of admissible contests contains all multistage (and single-stage) contests. Amegashie (1998) studies shortlisting of contestants, whereby a contest can only include a subset of potential candidates who compete for the right to enter the contest prior to the contest itself, focusing on the conditions under which shortlisting can reduce/increase the amount of resources spent. Thus, shortlisting constitutes a special case of our multistage contest with only two stages. This restricted framework, however, is unable to address the issue of the optimal number of stages in a contest, which is a central element of the contest design here.

2 Ingredients of a contest

In the contests considered below, we assume that n players expend efforts in an attempt to win a prize of V.[7] Each player's payoff is the same: the expected value of the prize less the amount of own effort, where the winning probabilities depend on the players' efforts.[8] The contest may consist of a series of single stage contests. We assume that in a single stage of a contest with j contenders for a prize, these probabilities are given by the Tullock (1980) contest success function:[9]

$$p_i(b_1, ...b_j) = \frac{b_i^\gamma}{\sum_{h=1}^{j} b_h^\gamma} \tag{1}$$

The parameter γ can be interpreted as a measure of how decisive relative contest effort is, or how discriminatory the contest rules are. For instance, when γ tends to zero, the probabilities in (1) tend to $1/j$, indicating that the designation of the winner is independent of effort. When γ tends to infinity, the contest becomes fully discriminatory: the prize is always awarded to the contestant who exerts the most effort, and the random elements of the outcome disappear.

Contestant i's payoff in such generic contest is

$$\frac{Mb_i^\gamma}{\sum_{h=1}^{j} b_h^\gamma} - b_i, \tag{2}$$

if M is the prize awarded in this single-stage contest. To ensure existence of pure strategy equilibria in each such contest we impose the restriction $0 < \gamma \leq j/(j-1)$. Then the equilibrium amount of individual effort can be directly derived:

$$b_i = b = \gamma M(j-1)/j^2 \tag{3}$$

[7] We assume in this paper that the contest rules, specifying in particular the winning odds, are exogenously given and focus exclusively on contest matching structure.

[8] This implicitly assumes that the marginal cost of effort is 1; the results can be easily extended, however, to allow any constant marginal cost.

[9] See Skaperdas, 1996, for its axiomatization.

Consider now the general (multi-stage) contest among n participants for the prize of V. In the first round, they are divided into several groups and compete within each group by expending effort. The winner in each group - who is determined according to (1) - goes up to the next round, where the contestants are again grouped into competition groups, etc. An optimal contest should determine the number of rounds and the individual grouping in each round, whereby contest structure is evaluated from the perspective of the organizer whose objective is maximization of effort. Letting A_k denote the total effort level in the kth round, the organizer's objective, therefore, is maximization of

$$E = A_1 + ... + A_L \tag{4}$$

where L is the number of rounds.

3 Effort maximizing contests

We start the analysis of the optimal (effort maximizing) contest structure with the following observation.

Lemma 1. *For any given number of rounds, the group size within each stage should be equal across the competing groups.*

Proof. Suppose that N individuals compete in a single round to win a prize whose value we normalize to 1. Let them be divided into r (exogenously given) groups, where g_j, $j = 1, ..., r$, is the size of group j. The prize is awarded with equal probabilities to the r winners of this round. Hence, the expected prize for each winning individual is $1/r$, and previous considerations allow us to establish that the amount of effort each group will expend, the probabilities of winning being as in (1), is $\gamma(1/r)(g_j - 1)/g_j$. Summing up, we obtain that total effort is $\gamma(1/r)\sum_{j=1}^{r}\frac{(g_j-1)}{g_j}$ which is maximized (subject to $g_1 + ... + g_r = N$) when $g_1 = ... = g_r$. $\square$

Let therefore g_k, $k = 1, ..., m$, be the size of each of the competition groups in the kth last round, and let α_k denote the dissipation rate in this round, that is, the ratio between the amount of contest effort and the size of the prize. Then total effort expended in the final is $\alpha_1 V$, leaving an undissipated rent of $(1 - \alpha_1)V$. Each of the finalists has the same chances of winning this rent, whose expected value for them is $(1 - \alpha_1)V/g_1$, implying that in all semifinals a total effort of $\alpha_2(1-\alpha_1)V$ is expended, etc. Thus proceeding backwards we obtain that total effort is given by:

$$E = [\sum_{k=1}^{m} \alpha_k \prod_{i=0}^{k-1}(1-\alpha_i)]V \tag{5}$$

with

$$\alpha_k = \gamma(g_k - 1)/g_k, \ k = 1, ...m, \text{ and } g_0 \equiv 1, \text{ so that } \alpha_0 = 0 \tag{6}$$

Furthermore, since in each competition group a single contestant emerges as a winner, who then proceeds to the next round, we obtain that multiplying the number of individuals in a group across rounds gives us the total number of individuals:

$$n = \prod_{k=1}^{m} g_k \tag{7}$$

The problem now is to choose the size of a group in each round, g_k, $k = 1, ..., m$, so as to maximize (5) subject to (6) and (7). Note that the objective function (5) is concave with respect to α_k, $k = 1, ..., m$. Furthermore, the composite function

$$F(g_1, ..., g_m) = [\sum_{k=1}^{m}(\gamma(g_k - 1)/g_k) \prod_{i=0}^{k-1}(1-(\gamma(g_i - 1)/g_i))]V \tag{8}$$

that is obtained from (5) by substitution of (6) is continuous, whereas its domain is compact, ensuring existence of a solution to the maximization problem.

The following lemma constitutes an important step to finding the solution.

Lemma 2. *For a given number of rounds, the optimal group size is the same across rounds.*

Proof. We form the Lagrangean:

$$L(g_1,...,g_m) = F(g_1,...,g_m) - \lambda(\prod_{j=1}^{m} g_j - n) = \tag{9}$$

$$[\sum_{k=1}^{m}(\gamma(g_k-1)/g_k)\prod_{i=0}^{k-1}(1-(\gamma(g_i-1)/g_i))]V - \lambda(\prod_{j=1}^{m} g_j - n)$$

Differentiating with respect to g_k we obtain after some manipulations the following first order conditions:

$$\gamma V \frac{\prod_{i=1}^{m}(1-(\gamma(g_i-1)/g_i))}{g_k(1-(\gamma(g_k-1)/g_k))} = \lambda n, \quad k = 1,...,m \tag{10}$$

Inspection of the system of equations in (10) reveals the solution of $g_k = g_{k+1} = g$, implying that the group size should be the same in all rounds of the contest.□

To examine the implications of this result for the optimal number of rounds, we note that the constraint in (7) is now rewritten as follows:

$$n = g^m \tag{11}$$

and the objective function (8) has the form

$$F(g,...,g) = \gamma V((g-1)/g)\sum_{i=0}^{m-1}[1-\gamma(g-1)/g]^i = V\{1-[1-\gamma(g-1)/g]^m\} \tag{12}$$

Hence the optimal number of rounds is the solution of minimizing

$$[1-\gamma(g-1)/g]^m \tag{13}$$

subject to (11). Substituting (11), the sign of the derivative of (13) with respect to m is the same as the sign of

$$[1 - \gamma(g-1)/g]\ln[1 - \gamma(g-1)/g] + \gamma \ln g/g \tag{14}$$

where g is given by (11). Note that when $\gamma = 0$ and when $\gamma = 1$, (14) equals zero irrespectively of m. Furthermore, (14) is a convex function of γ and it increases when $\gamma = 1$. Hence, when $\gamma = 1$, the different contest structures are equivalent and induce the same total effort. Also, (14) is positive when $\gamma > 1$ and is negative when $\gamma < 1$ implying that the optimal number of rounds is the smallest possible (that is, one) when $\gamma > 1$ and is the largest possible when $\gamma < 1$.

Hence, the simultaneous (S-) contest of all participants is the effort maximizing contest structure in the former case, and the pairwise multistage (T-, for tree) contest is the effort maximizing structure in the latter case (see Figure 1 which depicts an S- contest and a T-contest, each with 8 contenders).

[FIGURE 1 HERE]

To sum up,

Proposition 1. *When $\gamma = 1$, all different contest structures are equivalent provided that the group size is the same in each round of a contest; when $\gamma > 1$, the S-contest is the effort maximizing contest structure; and when $\gamma < 1$, the T-contest emerges as the effort maximizing structure.*

Table 1 illustrates the performance, in terms of the elicited effort, of the optimally adjusted contest structure, comparing a T-contest and an S-contest, for different values of the parameters.

[TABLE 1 HERE]

In the rent seeking literature, the extent of rent dissipation - in particular, the issue whether or not rent dissipation is complete for a large number of contestants - has always been a subject of intense controversy. It was argued that the answer to this question hinges on the nature of the contest success function: if it is sufficiently non-discriminatory (i.e., when $\gamma < 1$), then rent dissipation will be incomplete. Our results show that this conclusion, while correct when applied to S-contests, is incorrect when more flexibility in designing contests is allowed, because when n tends to infinity the effort level induced by the T-contest converges to V implying complete rent dissipation. This can be confirmed from (12), which tends to V as n tends to infinity. Hence, we obtain

Proposition 2. *When the contest structure is optimally adjusted, then* *complete rent dissipation is achieved for a large number of contestants, independently of the discriminatory extent of the contest technology.*

Table 1 above illustrates the power of a T-contest for $\gamma < 1$, showing the speed of convergence to full rent dissipation for different values of n.

4 Concluding remarks

This paper studies the issue of optimal structure of matches in contests. The significance of this study comes from the observation that contests may have multiple rounds, with winners of the rounds competing with each other. The design of a contest, therefore, should clearly address the issue of multistage contests.

While this paper focuses on certain aspects of contest design, many more issues are left for consideration in future research. For instance, we have assumed identical individuals. In the world of heterogeneous individuals differing with respect to their tastes and/or abilities, these differences should be taken into account when thinking about contest design. Risk aversion - both of the contestants and the organizer - may also be an important factor. For example, when the contestants are risk averse, big contests with a great deal of strategic uncertainty may be inferior to a series of smaller multistage contests. Finally, while the prize here has been assumed indivisible, an alternative assumption would introduce new considerations into the optimal design such as the allocation of the prize in the different rounds of the contest to the respective winners in these rounds.

We believe, however, that these extensions, while valuable and worth pursuing are unlikely to change the main messages of this paper. To sum them up in a nutshell our results indicate that the effort maximizing structure of a contest hinges on the extent of the decisiveness of a contest - that is, on the relative importance of one's effort to win the prize versus random factors. If the individual effort is sufficiently important, then simultaneous contests induces the larger total effort of the participants, and if random factors dominate, then a multistage contest which consists of pairwise matches does so. These results imply in particular that when contest structure can be optimally adjusted, then full rent dissipation is attained at the limit, independently of the discriminatory nature of the contest success function. This conclusion contradicts the view derived from modeling simultaneous contests that the extent of rent dissipation hinges on the success technology as embodied in the contest success function.

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