

# Beliefs in Repeated Games

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# MOTIVATION

Social dilemmas capture many situations of interest: Cournot competition, public good provision, natural resource extraction, *etc.*

Key tension: individually rational vs socially optimal.

Long extensive theoretical literature.

Recent experimental literature: how cooperation varies with

- ▶ payoffs,
- ▶ monitoring,
- ▶ termination ( $H$  and  $\delta$ ).

# MOTIVATION

Two premises underlie the equilibrium predictions:

- ▶ beliefs are correct, and
- ▶ actions are best responses to those beliefs.

In repeated interactions, this involves

- ▶ many contingencies, complex strategies,
- ▶ and sometimes multiple equilibria.

→ Making it challenging to form correct beliefs and to best respond.

**Goal.** Study beliefs to better understand cooperation in the repeated *Prisoners' Dilemma* (PD), comparing finite and indefinite games.

# MOTIVATION

## QUESTIONS

1. Are *round* beliefs accurate?
2. Is the relation between choices and beliefs different in the finite and indefinite games?
3. Can strategy choice be rationalized by beliefs?

# MOTIVATION: IMPLEMENTATION

THE STUDY OF REPEATED GAMES AND OF BELIEFS PRESENTS CHALLENGES

Simple repeated games:

- ▶ deterministic,
- ▶ with perfect monitoring.

Simple belief elicitation:

- ▶ elicit beliefs over other's choice in current round,
- ▶ recover beliefs over strategies econometrically.

# DESIGN

2 treatments (between subjects):

**Finite:** Stage game repeated for 8 rounds.

**Indefinite:** Stage game repeated with probability  $7/8$ .

- Block Random Design for the first 8 rounds.

Random matching between supergames.

Sessions end with the first supergame after 1 hour of play.

Belief elicitation:

- ▶ Beliefs about other's action in the current round.
- ▶ Introduced in the 5th supergame.
- ▶ BSR: Hossain-Okui (2013) modified by Wilson-Vespa (2018).

Risk preferences at the end:

- ▶ Bomb task: Crosetto and Filippin (2013).

# DESIGN

Table: STAGE GAME (in ECU)

	C	D
C	51, 51	22, 63
D	63, 22	39, 39

Treatment	Session	Supergames			
		Action	Actions and Beliefs		
		Only	Early		Late
Finite	1	1, 2, 3, 4	5, 6, 7	8, 9	10, 11, 12
	2	1, 2, 3, 4	5, 6, 7	8, 9	10, 11, 12
	3	1, 2, 3, 4	5, 6, 7	8, 9, 10	11, 12, 13
	4	1, 2, 3, 4	5, 6, 7	8	9, 10, 11
	5	1, 2, 3, 4	5, 6, 7	8, 9, 10	11, 12, 13
	6	1, 2, 3, 4	5, 6, 7	8, 9, 10	11, 12, 13
	7	1, 2, 3, 4	5, 6, 7	8, 9	10, 11, 12
	8	1, 2, 3, 4	5, 6, 7	8, 9	10, 11, 12
Indefinite	1	1, 2, 3, 4	5, 6, 7		8, 9, 10
	2	1, 2, 3, 4	5, 6		7, 8, 9
	3	1, 2, 3, 4	5		6, 7
	4	1, 2, 3, 4	5		6, 7, 8
	5	1, 2, 3, 4	5, 6, 7	8, 9	10, 11, 12
	6	1, 2, 3, 4	5		6
	7	1, 2, 3, 4	5, 6, 7		8, 9, 10
	8	1, 2, 3, 4	5, 6		7, 8, 9

14 to 20 subjects per session: **302 subjects** in total.

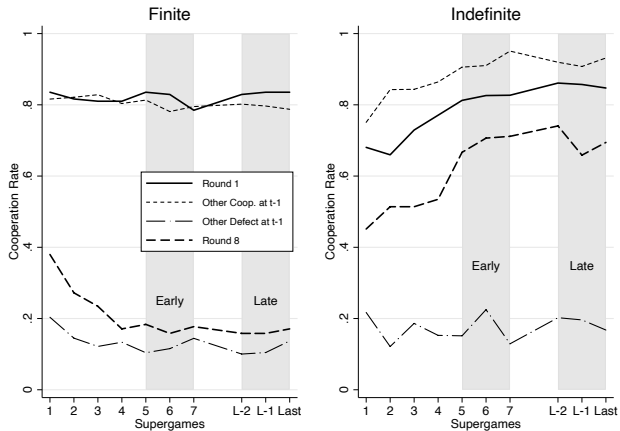
Payment: \$8 + choices from two supergames (pre/post) + beliefs in one.

Earnings from \$22.00 to \$63.75 (with an average of \$35.30).



# ACTIONS

# COOPERATION OVER SUPERGAMES



**Result 1***(Replication / Design Neutrality)***Reproduce *typical* qualitative data **patterns**.**

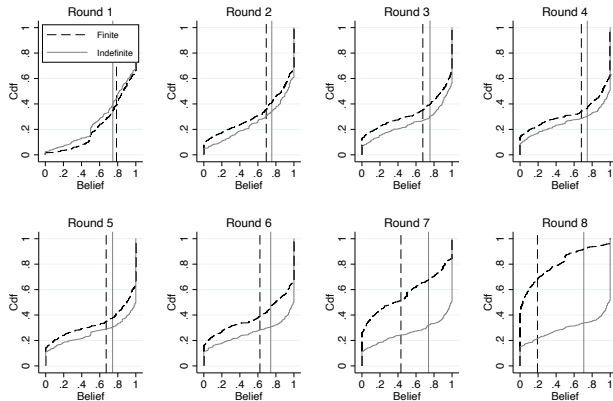
Confirms that:

- (i) **Cooperation is history-dependent** in both games.
- (ii) **Cooperation evolves differently** in Finite and Indefinite games.

**KEY DIFFERENCE:** cooperation collapses at the end only in Finite games.

# **BELIEFS**

# DISTRIBUTION OF BELIEFS BY TREATMENT AND ROUND



Late supergames.  
Vertical lines indicate respective means.

## Result 2

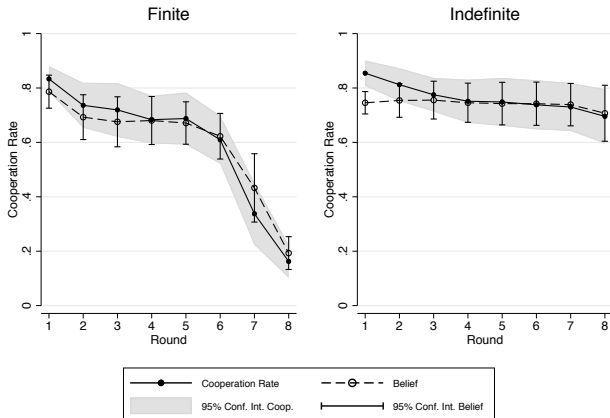
*(Question 2)*

**Beliefs are different** in Finite and Indefinite games.

KEY DIFFERENCE: Beliefs collapse at the end of Finite games.

# ACTIONS AND BELIEFS

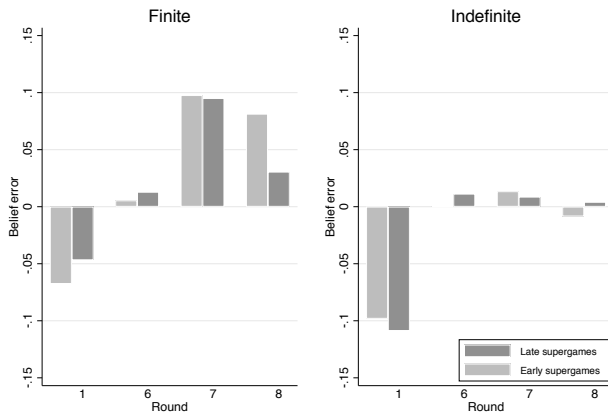
# ACTIONS AND BELIEFS



Late supergames.

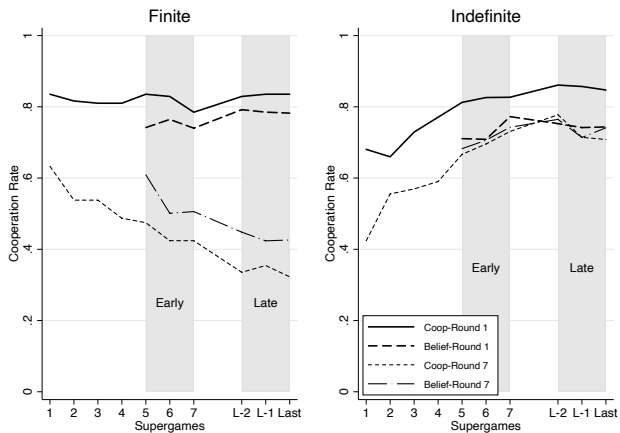


# BELIEFS ERRORS IN EARLY VS. LATE SUPERGAMES



Belief error denotes average difference between beliefs and actions.

# BELIEFS ERRORS: ROUNDS 1 AND 7



### Result 3

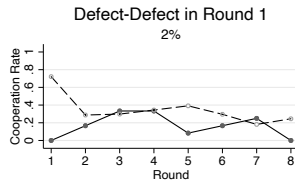
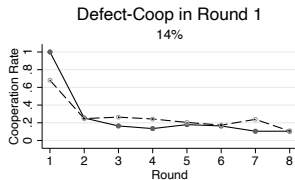
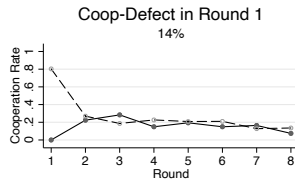
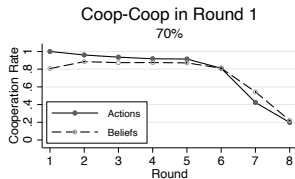
*(Question 1)*

Overall, average **beliefs are accurate**.

However, some systematic deviations are present and persistent.

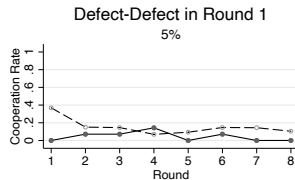
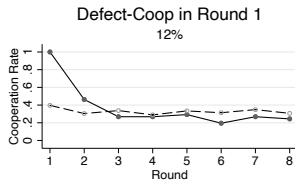
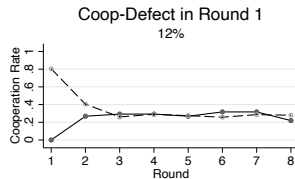
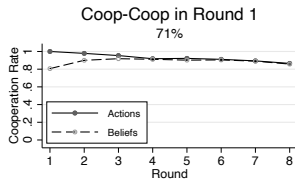
**KEY DIFFERENCE:** end-of-game optimism in Finite games and early pessimism in Indefinite games.

# CONDITIONAL BELIEFS: FINITE



Own choice listed first.  
Late supergames.  
Percentage of cases under the title.

# CONDITIONAL BELIEFS: INDEFINITE



Own choice listed first.  
Late supergames.  
Percentage of cases under the title.

# ACCURACY

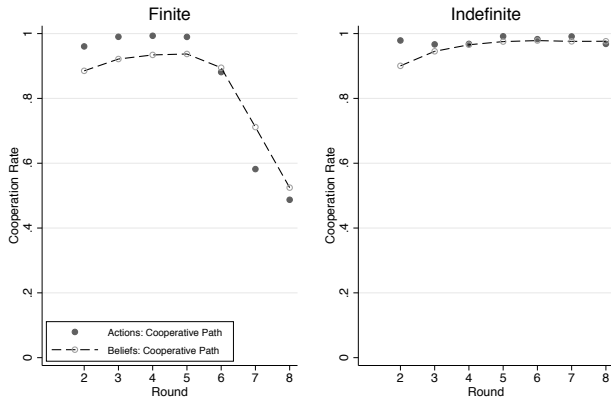
# ACCURACY

		Finite			Indefinite		
		Correct	Within		Correct	Within	
		Tercile	10%	5%	Tercile	10%	5%
	<u>Round 1</u>	73	14	7	67	10	5
	<u>Round 2</u>						
	CC	91	60	9	91	66	58
Round 1	CD	67	16	9	29	10	2
Actions	DC	66	7	7	56	17	12
	DD	67	8	8	79	0	0
	Average	83	45	9	80	52	45

Round 1 actions are listed own action first, other action second: i.e.  $(a_i, a_j)$ .

Average is weighted by the number of observations.

# COOPERATIVE PATH



Cooperative path: joint cooperation up to that round.  
Late supergames.



## Result 4

(Questions 1 and 2)

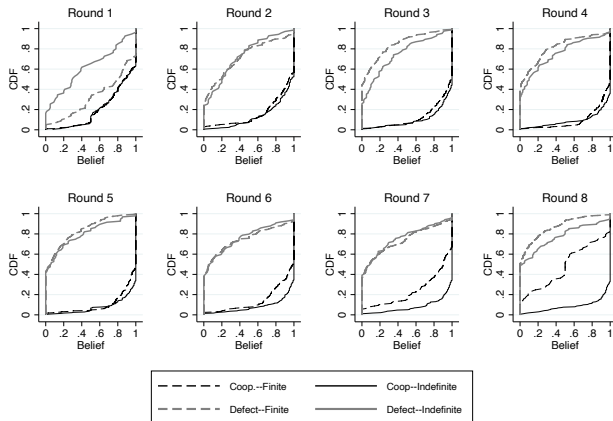
(i) **Beliefs respond to actions.**

(ii) **Given the same history, beliefs are different** across games.

→ History of play can't account for all the variation in beliefs: *Supergame strategies*.

**KEY DIFFERENCE:** subjects (correctly) anticipate defection in Finite games despite a history of joint cooperation.

# BELIEFS BY ACTION AND TREATMENT: ROUNDS 1-8



Late supergames.

## Result 5

(Questions 1 and 2)

Beliefs correlate to actions, more optimistic subjects are more likely to cooperate.

The **same round belief generates different actions** in each game.

→ Actions are determined by beliefs AND game: *Supergame strategies*.

# **BELIEFS OVER STRATEGIES**

# ESTIMATION STRATEGY (INTUITION)

Data			Beliefs Over Strategies For Player 1
	Round		
	1	2	
$a_1$			$\tilde{p}(AD) =$
$a_2$			$\tilde{p}(AC) =$
$\mu_1$			$\tilde{p}(G) =$

# ESTIMATION STRATEGY (INTUITION)

Data		
Round		
	1	2
$a_1$		
$a_2$		
$\mu_1$	0.60	

Beliefs Over Strategies  
For Player 1

$$\tilde{p}(AD) =$$

$$\tilde{p}(AC) =$$

$$\tilde{p}(G) =$$

## ESTIMATION STRATEGY (INTUITION)

	Data	
	Round	
	1	2
$a_1$		
$a_2$		
$\mu_1$	0.60	

Beliefs Over Strategies For Player 1		
$\tilde{p}(AD)$	=	0.40
$\tilde{p}(AC)$	=	?
$\tilde{p}(G)$	=	?

## ESTIMATION STRATEGY (INTUITION)

Data			Beliefs Over Strategies For Player 1	
	Round			
	1	2		
$a_1$	D		$\tilde{p}(AD)$	= 0.40
$a_2$	C		$\tilde{p}(AC)$	= ?
$\mu_1$	0.60		$\tilde{p}(G)$	= ?



# ESTIMATION STRATEGY (INTUITION)

	Data	
	Round	
	1	2
$a_1$	D	
$a_2$	C	
$\mu_1$	0.60	0.10

Beliefs Over Strategies For Player 1		
$\tilde{p}(AD)$	=	0.40
$\tilde{p}(AC)$	=	?
$\tilde{p}(G)$	=	?

# ESTIMATION STRATEGY (INTUITION)

	Data	
	Round	
	1	2
$a_1$	D	
$a_2$	C	
$\mu_1$	0.60	0.10

Beliefs Over Strategies For Player 1		
$\tilde{p}(AD)$	=	0.40
$\tilde{p}(AC)$	=	0.06
$\tilde{p}(G)$	=	0.54

## ESTIMATION STRATEGY (INTUITION)

Data			Beliefs Over Strategies For Player 1		
Round					
	1	2			
$a_1$	D		$\tilde{p}(AD)$	=	0.40
$a_2$	C		$\tilde{p}(AC)$	=	0.06
$\mu_1$	0.60	0.10	$\tilde{p}(G)$	=	0.54

- + Allow for 1 to believe 2 implements strategy with errors ( $\tilde{\beta}$ ).
- + Allow for 1 to report with some error ( $\nu$ ).
- + Estimate beliefs separately for each strategy type.

# ESTIMATION STRATEGY

1. *Estimate strategies at the population level.*
2. *Use these estimates to strategy-type each subject.*
3. *Estimate beliefs over strategies separately for each type.*

# ESTIMATION STRATEGY

1. *Estimate strategies at the population level.*
  - Use SFEM (Dal Bó and Fréchette 2011).
  - Finite mixture with 16 strategies (Fudenberg *et al.* 2012, Embrey *et al.* 2018):  
AD, AC, Grim, Grim2, Grim3, TFT, TF2T, 2TFT, STFT, T2–T8.
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# ESTIMATION STRATEGY

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## 2. *Use these estimates to strategy-type each subject.*

- Use SFEM results + subject actions (and history) + Bayes' rule.
- Associate subject to strategy with highest Bayesian posterior: the *strategy type*.

## 3. *Estimate beliefs over strategies separately for each type.*

### Strategy Prevalence and Typing

Type	Finite		Indefinite		
	Share		Type	Share	
	SFEM	Typing		SFEM	Typing
<b>T7</b>	0.30	0.35	<b>TFT</b>	0.34	0.58
<b>T8</b>	0.22	0.20	<b>Grim</b>	0.15	0.07
AD	0.12	0.12	AC	0.10	0.10
TFT	0.09	0.12	<b>AD</b>	0.09	0.10
T6	0.08	0.08	TF2T	0.09	0.03
Grim	0.07	0.02	Grim2	0.07	0.02
TF2T	0.03	0.04	Grim3	0.06	0.02
Grim3	0.03	0.03	2TFT	0.05	0.01
STFT	0.02	0.02	STFT	0.04	0.04
AC	0.02	0.01	T3	0.02	0.03
Grim2	0.01	0.01	T8	0.01	0.01
T2	0.01	0.01	T7	0.00	0.00
2TFT	0.00	0.00	T6	0.00	0.00
T5	0.00	0.00	T5	0.00	0.00
T4	0.00	0.00	T4	0.00	0.00
T3	0.00	0.00	T2	0.00	0.00

Estimation using late supergames.  
 SFEM estimate for  $\beta$  are 0.94 for both.

## Result 6

There is **strategic heterogeneity** within and across treatments.

KEY DIFFERENCE: in Finite games subjects mostly use threshold strategies and in Indefinite games subjects mostly use conditionally cooperative strategies.



# ESTIMATION STRATEGY

## 1. *Estimate strategies at the population level.*

- Use SFEM (Dal Bó and Fréchette 2011).
- Finite mixture with 16 strategies (Fudenberg *et al.* 2012, Embrey *et al.* 2018): AD, AC, Grim, Grim2, Grim3, TFT, TF2T, 2TFT, STFT, T2–T8.

## 2. *Use these estimates to strategy-type each subject.*

- Use SFEM results + subject actions (and history) + Bayes' rule.
- Associate subject to strategy with highest Bayesian posterior: the *strategy type*.

## 3. *Estimate beliefs over strategies separately for each type.*

- Belief over strategies + history of play + Bayes' rule pin down round beliefs.
- Assume that subjects report their beliefs with noise.
- Find beliefs over strategies  $\tilde{p}$ , implementation noise  $\tilde{\beta}$ , reporting noise  $\nu$  via MLE.

# BELIEFS OVER STRATEGIES

## FINITE

Type	Share		Estimated Beliefs - $\bar{p}$								$\nu$	$\bar{\beta}$
	SFEM	Typing	T8	T7	TFT	2TFT	GRIM	AD	TF2T	Other		
T7	0.30	0.35	0.43	0.43	0.00	0.00	0.14	0.00	0.00	0.00	0.04	1.00
T8	0.22	0.20	0.51	0.00	0.01	0.01	0.00	0.07	0.17	0.22	0.04	1.00
AD	0.12	0.12	[0.00]	0.00	0.00	0.76	[0.00]	0.23	0.00	0.00	0.06	1.00
TFT	0.09	0.12	0.30	0.00	0.55	0.00	0.00	0.11	0.00	0.04	0.05	1.00
T6	0.08	0.08	0.48	0.50	0.00	0.00	[0.00]	0.01	0.00	0.01	0.03	1.00
GRIM	0.07	0.02	0.23	0.00	0.26	0.00	0.00	0.17	0.05	0.28	0.07	1.00
Other	0.12	0.11	0.04	0.18	0.36	0.00	0.24	0.04	0.02	0.13		
All	0.00	0.00	0.33	0.19	0.12	0.09	0.07	0.07	0.04	0.09		

Estimation on late supergames out of 16 strategies: AD, AC, Grim, Grim2, Grim3, TFT, TF2T, 2TFT, STFT, T2-T8.

Rows, top 6 *played* strategies. Columns, top 7 *believed* strategies.

SFEM estimate for  $\beta$  is 0.94.

Estimates in [square brackets] are not estimated due to collinearity.

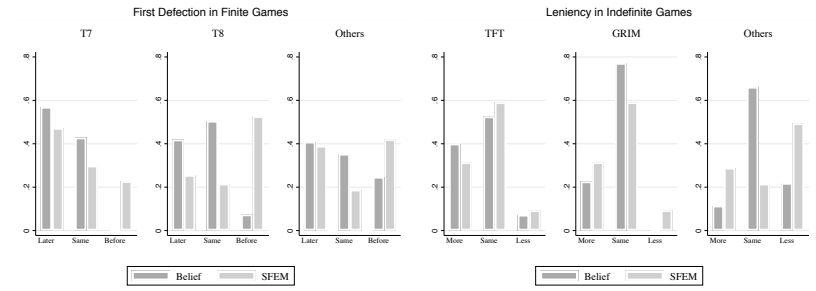
# BELIEFS OVER STRATEGIES

## INDEFINITE

Type	Share		Estimated Beliefs - $\tilde{p}$								$\nu$	$\tilde{\beta}$
	SFEM	Typing	Grim	TF2T	TFT	AC	AD	Grim2	STFT	Other		
TFT	0.34	0.58	0.21	0.14	0.27	0.08	0.07	0.16	0.04	0.02	0.01	1.00
Grim	0.15	0.07	0.74	0.08	0.03	0.09	0.00	0.06	0.00	0.00	0.06	1.00
AC	0.10	0.10	0.00	0.48	0.00	0.52	0.00	0.00	0.00	0.00	0.05	1.00
AD	0.09	0.10	0.07	0.00	0.01	0.01	0.90	0.01	0.00	0.01	0.04	1.00
TF2T	0.09	0.03	0.09	0.91	0.00	0.00	0.00	0.00	0.00	0.00	0.01	1.00
Grim2	0.07	0.02	0.00	0.24	0.00	0.24	0.00	0.52	0.00	0.00	0.05	1.00
Other	0.16	0.10	0.15	0.09	0.25	0.09	0.14	0.11	0.02	0.17		
All			0.22	0.22	0.14	0.13	0.13	0.12	0.02	0.03		

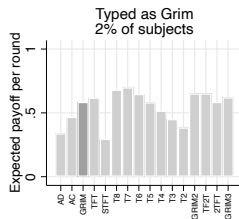
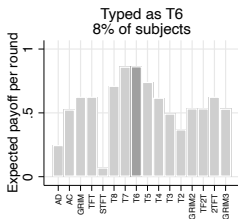
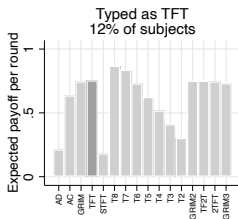
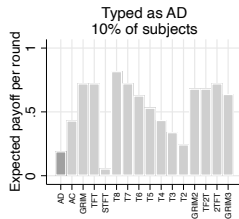
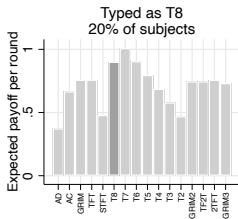
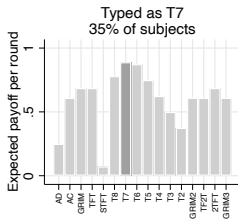
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 Rows, top 6 *played* strategies. Columns, top 7 *believed* strategies.  
 SFEM estimate for  $\beta$  is 0.94.

# BELIEFS VS REALITY



# BEST RESPONSE IN FINITE GAME: TOP 6

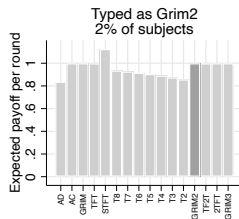
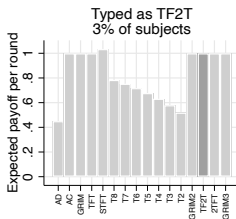
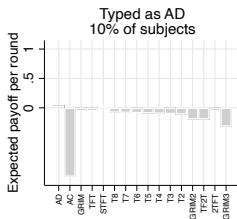
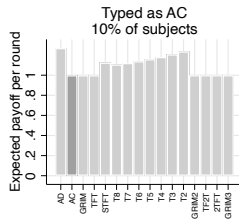
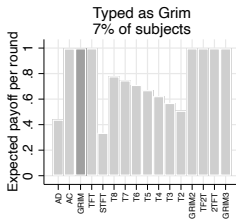
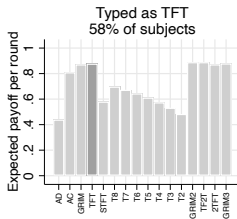
## Finite



The strategy corresponding to the type is highlighted in dark grey.  
Analysis uses normalized stage-game payoffs.

# BEST RESPONSE IN INDEFINITE GAME: TOP 6

## Indefinite



The strategy corresponding to the type is highlighted in dark grey.  
Analysis uses normalized stage-game payoffs.

## Result 7

*(Question 3)*

Subjects using **different strategies** have **different beliefs**.

Of interest:

- (i) **Expect the correct type of strategies**, threshold strategies in Finite games and conditionally cooperative strategies in Indefinite games.
- (ii) **Tendency to think others will not defect before you.**
- (iii) Vast majority of **subjects** (close to) **best respond** to their beliefs.

# CONCLUSION

- ▶ *Round beliefs* are remarkably **accurate** and **forward-looking**.
- ▶ The **same action** in both games is supported by **different beliefs**.
- ▶ **Beliefs over strategies** are very **different across games**. In both cases:
  - **Bias**: Underestimate the likelihood that others move to defection earlier than they do.
  - **Subjectively Rational**: Many strategies are (close to) best responses to supergame beliefs.



# BELIEF ELICITATION IMPLEMENTATION

In each round of a match, after you make a choice, we will ask you to submit your belief about the choice of the person you are paired with.

To indicate your beliefs, you will use a slider. Where you move the slider will represent your best assessment of the likelihood (expressed as chance out of 100) that the person you are paired with chose **1** or **2**.

[...] The belief that you report in that round will determine your chance of winning a prize of 50 points.

To determine your payment, the computer will randomly draw two numbers. For each draw, all numbers between 0 and 100 (including decimal numbers) are equally likely to be selected. Draws are independent in the sense that the outcome of the first draw in no way affects the outcome of the second draw.

If the person you are paired with chose **1** in that round and the number you indicated as the likelihood that the other chose **1** is larger than either of the two draws, you will win the prize.

If the person you are paired with chose **2** in that round and the number you indicated as the likelihood that the other chose **2** is larger than either of the two draws, you will win the prize.

The rules that determine your chance of winning this prize were purposefully designed so that you have the greatest chance of winning the prize when you answer the question with your true assessment on how likely the person you are paired with chose **1** or **2**.

